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A Historical Essay on the Scientific School of V.A. Yakubovich

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Abstract—The milestones of the history of the scientific school on cybernetics (the School), established in 1959 by outstanding scientist V.A. Yakubovich at Leningrad State University (LSU), are presented. The connections of the School with other Russian and foreign scientific schools in related fields are outlined.

Keywords: history, cybernetics, control theory, St. Petersburg State University, Department of Theoretical Cybernetics

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This paper describes the main milestones in the history of the scientific school of cybernetics and control theory (the School), established in 1959 by outstanding scientist V.A. Yakubovich at Leningrad State University (LSU). The School will celebrate its 65th anniversary in 2024. The essay is partially based on the publications [1–3] on the history of the Department of Theoretical Cybernetics, St. Petersburg State University (SPbSU), as well as its scientific directions and related issues. The authors are not intended to provide a complete bibliographic survey of the School's results concerning the aspects of its activities touched upon in the paper, particularly due to its limited scope. The authors present either key works or illustrative and sometimes subjectively selected examples of research works on a certain topic. The authors apologize to colleagues whose publications are not mentioned below.

The beginning of the history of cybernetics at St. Petersburg (Leningrad) University can be considered the year 1956, when Vladimir Andreevich Yakubovich, a 30-year-old candidate of physics and mathematics, came to the Faculty of Mathematics and Mechanics. It was a time of great changes in society and in science, the beginning of the thaw. The first electronic computing machines (ECM) and publications rehabilitating cybernetics [4, 5] appeared. Cybernetics was gaining popularity, and lectures and discussions about it spread everywhere. The country's first section of cybernetics was established at the Leningrad House of Scientists; it was headed by academician and future Nobel laureate L.V. Kantorovich. The Computing Center (CC) and research laboratories were organized in Leningrad University to master and use the new (fantastic, as it seemed at that time) capabilities of computers. Following the impact of the seminal book by N. Wiener [6], cybernetics was perceived primarily as a scientific foundation for the application of computer technology and automatic devices. Not surprisingly, when the administration of the Faculty of Mathematics and Mechanics proposed to V.A. Yakubovich to gather a group of researchers in the field of advanced mathematical methods of automation and control systems, the "cybernetic flag" became most suitable for the group. Thus, in 1959, the Laboratory of Theoretical Cybernetics (LTC, the Laboratory) appeared in the CC of LSU.

The first years of LTC research focused on pattern recognition and machine learning. The Laboratory developed and generalized Rosenblatt's concept of perceptron, which was popular at that time, and several approaches to the mathematical theory of pattern recognition [7–9].[1] A series of applied problems were successfully solved, including handwriting and aerial survey photo recognition, extraction of useful signals from noisy material, and automatic description and analysis of scenes [11–14]. The Laboratory's team owns a series of original algorithmic solutions for the entire problem and its individual aspects, such as B.N. Kozinets' algorithm for memory-saving class separation [15, § 2.6], [16, Ch. 6], A.A. Schmidt's method of algebraic invariants in image recognition problems [16, Ch. 8], and others. Comprehension of the ideas accumulated at that stage led V.A. Yakubovich to the general concept of an infinite a priori unknown recursive system of inequalities, where inequalities are added step-by-step in real time, and finitely convergent algorithms for solving such systems in real time [17]. Later on, this concept and related methods were repeatedly shown to be productive in various fields. The key approach to solving such systems developed in LTC was subsequently called *the method of recursive aim inequalities* [18].

The emergence of a new field— cybernetics—inevitably gave rise to a discussion of its relationship with the traditional theory of automatic control. A fruitful channel for that discussion was paved, among others, by the notion of adaptability, i.e., the autonomous capability of a system to adjust successfully to a priori essentially uncertain conditions of its operation (external and internal). In those years, statistical approaches to adaptive control prevailed in the Moscow school. In particular, Ya.Z. Tsypkin elaborated the theory of adaptive and learning systems based on statistical estimation and stochastic approximation methods [19, 20]. In parallel, V.A. Yakubovich developed an original alternative (deterministic) approach without involving probability theory; a key element of his approach is the method of recursive aim inequalities. V.A. Yakubovich gave historically the first general mathematical definition of an adaptive system [21, 22]. The basic material on the theory of recursive aim inequalities and adaptive control can be found in the monographs [15, 23]; a survey of subsequent works is available in [24–26].

The results of V.A. Yakubovich's team in the field of adaptive systems were naturally continued in robotics research. Initially, scientists all over the world carefully avoided the term "robot" and its derivatives, believing them to be frivolous and suitable for science fiction at most. There are grounds to state that V.A. Yakubovich pioneered "robot" as a now generally recognized scientific concept; see his paper [21] published in *Doklady Akademii Nauk USSR.* The method of recursive aim inequalities was used therein to solve the problem of self-learning of a manipulator robot (an "eye–arm" robot) and to prove several theorems about "the rationality of robots" in the sense of the definition introduced.

In almost all industrialized countries, the late 1960s and early 1970s were marked by the rapidly growing interest in production automation based on manipulator robots with elements of artificial intelligence. On the crest of that wave, in 1973, a robotics group was formed in LTC, headed by V.A. Yakubovich's students—Dr. Sci. (Eng.) A.V. Timofeev [27–29], and then Cand. Sci. (Phys.–Math.) S.V. Gusev [30, 31]. Note the main robotics achievements of LTC during that period: a mathematical theory of adaptive robots and a theory to train them to complex rational behavior [32–34]. The viability of this theory was first demonstrated by vivid examples of solving prototypical problems, such as the problem of training a robot to ride a two-wheeled bicycle, as well as training other adaptive robots. (As pets in the team they received nicknames "grasshopper," "hawk," "eye–arm," and others.) The significance of the solved problems is emphasized by the fact

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[1] The paper [7] was actually the first work in Russian devoted to machine learning. It was reprinted and translated into English in 2021; see [10].

that the corresponding results in 1972 were selected by the international organizing committee for presentation at the IFAC World Congress (Paris, 1972); see [35]. (It was joked that V.A. Yakubovich went to Paris on a bicycle as a speaker.) In the future, the effectiveness of the theory developed in the team was demonstrated by experiments (one of the first in the country) with real wheeled robots [30], started in 1974; in 1980, they were continued using a more advanced experimental robot developed in LTC [36]. In the 1980s, the LTC robotics group participated in the development of a manipulator control system for Buran, the reusable space shuttle project. In the 1970s–1980s, the LTC team also elaborated the theory of adaptive control of robotic systems described by general Lagrange equations [37–40]; those studies were pioneering in many respects and underlie the subsequent large-scale development of the corresponding research area in the world.

The rapid development of cybernetics and control theory in the 1960s led to the emergence of numerous algorithms for control, adaptation, recognition, learning, estimation, and filtering. The need arose to generalize the results obtained and to unify the algorithms proposed, i.e., to identify their key ideological core. Probably, Ya.Z. Tsypkin was the first to feel that need [19, 20]: he proposed to treat various problems of recognition, estimation, control, etc. as problems of minimizing the mean of a certain loss function. As a result, the chaotic mass of then-existing disparate algorithms was represented in the form of systematized special cases of uniform probabilistic gradient iterative procedures for minimizing or estimating parameters. However, the basic adaptation (self-learning) and control algorithms related to the continuous-time case did not fit into that scheme. Following analysis attacks from different directions, it gradually became clear that the algorithms mentioned can be unified within Ya.Z. Tsypkin's scheme by passing from the gradient of the objective function to the gradient of the rate of its change along the trajectories of the controlled object. Probably, the most general and complete approach to implementing that idea was proposed and developed by A.L. Fradkov [41] and named by him *the speed gradient method*.

Initially, the method was mainly focused on adaptive control and identification problems. As a result of subsequent many-year research, it was developed and applied as a universal approach to designing various continuous dynamic systems in mathematics, physics, engineering, biology, and other fields. For example, based on this approach, control and synchronization problems were solved for a wide class of oscillatory, including chaotic, systems. The corresponding results opened new perspectives in vibration engineering, laser and chemical technologies, and information transmission systems. Due to practical simplicity and the availability of a rigorous mathematical justification of the obtained algorithms, this method became generally recognized as a research tool, both in the USSR (Russia) and abroad. The number of publications where the method is applied in one form or another has been constantly growing and now reaches several hundred. Recently, interest in the speed gradient method has increased as a tool for understanding the laws of evolution to comprehend better the dynamics of physical, biological, and other systems. In this form, the method is known as *the speed gradient principle* [42, 43].

In the early 1970s, a bionics group was established at LTC under the leadership of Dr. Sci. (Psy.) R.M. Granovskaya. The task of the group was to study and model the phenomena of perception and recognition, as well as memory mechanisms of living organisms, including humans [44, 45]. A considerable amount of experimental and theoretical studies was conducted, and the results obtained were largely motivated and actively implemented by interested organizations.

In 1970, the Department of Theoretical Cybernetics (DTC, the Department) was established on the basis of LTC. The three pioneering alumni of the Department—G.S. Aksenov, B.D. Lyubachevskii, and A.L. Fradkov—graduated soon, in 1971. LTC and DTC were in fact a single team, with common affairs and minimal influence of the formal distribution of employees between them. The LTC staff was engaged in teaching, while DTC members conducted research on topics common to LTC and very often in collaboration with LTC colleagues. Discussion of relevant components of that research was systematically transferred from the Laboratory walls to the classrooms: while still learning the professional base, students were exposed to the cutting edge of the field. For example, the LTC-DTC staff often presented new, as yet unpublished results in lectures. Sometimes students were given proofs of theorems that had been obtained only the day before. There were a sense of lively participation in mathematical creativity and a feeling of being at the forefront of science. Sometimes, students found inaccuracies in proofs or suggested ways to improve the considerations. Such students were thanked in publications, which caused a sense of pride and a desire to move on. Note that from long ago, the motto of the department has been Docendo discimus, which means "learning by teaching."

In addition to the purely cybernetic direction (recognition, machine learning, artificial intelligence, adaptive systems, robots, etc.), the field of scientific interests of the team has been covering several classical branches of mathematics and control theory. They concern linear differential equations, dynamic systems and parametric resonance (V.A. Yakubovich, V.N. Fomin, and V.I. Derguzov), stability and oscillations in nonlinear dynamic systems, including phase synchronization and frequency auto-tuning systems, stability and oscillations in pulse-modulated systems (G.A. Leonov, A.I. Shepeljavyi, A.Kh. Gelig, and A.N. Churilov), optimal control (A.S. Matveev, A.E. Barabanov, and V.A. Yakubovich), estimation and filtering theory (V.N. Fomin and A.E. Barabanov), and others.

Even before the establishment of the Laboratory and the Department, V.A. Yakubovich obtained fundamental results on the stability of linear systems of differential equations with periodic coefficients and parametric resonance. He proved I.M. Gelfand's hypothesis that in the functional space of coefficients of two-dimensional Hamiltonian systems, the set of coefficients corresponding to stable systems decomposes into a countable number of connected domains; moreover, he showed that the Lyapunov criterion, popular in the subject in those years, applies to one of them only. V.A. Yakubovich obtained stability criteria for each domain, which, like the mentioned Lyapunov criterion, are irreducible in a certain natural sense. These results were then transferred by V.N. Fomin and V.A. Derguzov to systems with an infinite-dimensional state space. The fundamental monograph [46] summarized the intermediate outcomes of this direction and is still actively cited in the works of mathematicians, physicists, and engineers.

Among the numerous scientific results of the team, perhaps the greatest fame and influence has been gained by the achievements related to the so-called "frequency theorem," also known as the Yakubovich–Kalman lemma and the Kalman–Yakubovich–Popov (KYP) lemma. It was proved by V.A. Yakubovich and first published in 1962 [47]. This theorem gives mathematically beautiful, transparent, and constructive conditions for the solvability of a rather complex system of relations, which is found in a variety of problems in stability theory, automatic control, robotics, and other fields; in turn, the solution of this system of relations is the key to solving the main problem and its qualitative analysis. The importance and authority of the frequency theorem are the derivatives of its productivity in a whole range of diverse fields and problems, where it has given a second wind to the method of Lyapunov functions. For example, it allowed obtaining a whole series of new constructive criteria for absolute stability, instability, auto-oscillations, and the existence of globally stable periodic and almost periodic modes in a variety of nonlinear systems, as well as advancing in the study of the so-called strange attractors of such systems and developing new optimal and adaptive control methods; some of these results were presented in 1978 in the monograph [48]. This book is still relevant and interesting to scientists of different countries, as evidenced, in particular, by the publication of its English translation in 2004. Moreover, the lemma under consideration allowed establishing a kind of exhaustive results, surely covering all the conditions of a given type of system behavior, that can be obtained using Lyapunov functions from popular classes (e.g., a quadratic form, a quadratic form plus the integral of a nonlinearity, etc.).

Note that the frequency theorem is sometimes called the Great Lemma of Systems Theory: it is "officially" recognized by the international scientific community as one of the cornerstones of modern control theory. For example, this fact is reflected by the presence of V.A. Yakubovich's paper on the frequency theorem [47] in *Twenty Five Seminal Papers in Control* (Wiley—IEEE Press, 2000), a special collection containing 25 papers with the greatest impact on the development of control theory in the 20th century according to the IEEE Control Systems Society.

At first, the frequency theorem was proved for control systems described by ordinary differential equations. Subsequently, it was extended in different directions, in particular, it was transferred to many other classes of controlled systems. Among them, note discrete-time systems, stochastic systems, adaptive systems, systems with an infinite-dimensional state space (e.g., described by partial differential equations, equations with delayed argument, differential equations in an infinite-dimensional Hilbert space, integral equations, etc.), and systems over ordered fields [49–59]. These achievements were overwhelmingly not an end in themselves but a road to a scattering of new self-sufficient results pushing the boundaries of understanding of relevant fields, e.g., to the criteria of absolute stability and instability for the classes of systems under consideration. In this scientific development, the school of V.A. Yakubovich went, mutually enriching, hand in hand with other scientific schools, e.g., with the Nizhny Novgorod school (V.A. Brusin, P.V. Pakshin, V.A. Ugrinovskii, etc.) [60–64]. The history and current state of this direction were described in detail in the surveys [65, 66] and the collective monograph [3].

Note that necessary and sufficient conditions for the existence of linear output-feedback of a linear system ensuring the existence of its quadratic Lyapunov function were obtained in [55, 56]. This property of the system is equivalent to its passivity, meaning the fulfillment of some dissipation-type inequality on the trajectories of the system. Therefore, the results [55, 56] can be termed passification theorems for linear systems. These statements underlaid a general approach to system design called the passification (passivation) method. Subsequently, the method was extended to a wide class of control and estimation problems for nonlinear and adaptive systems [67–70]. The passification method is now applied by researchers from various countries [71–73]. In Russia, it is actively used particularly in the scientific school of ITMO University (V.O. Nikiforov, A.A. Bobtsov, etc.) [74, 75, 141].

The wide applicability of the frequency theorem motivated V.A. Yakubovich to construct an abstract theory of absolute stability: using the apparatus of functional analysis, such a theory generalizes the mass of known results and also creates a comfortable basis for their extension to all new types of equations. Note that research on the frequency theorem is related to the now ultra-popular method of linear matrix inequalities (LMIs). Accordingly, the authors of the book [77] called V.A. Yakubovich the "father" of the scientific direction based on this method (in honorable company with "grandfather" A.M. Lyapunov). A lot of adherents in the world have been successfully developing this direction for a long time in a surprisingly wide range of applied fields.

The frequency theorem was born in the stability analysis of equilibria of nonlinear dynamic systems as an answer to the following question: under what conditions does there exist a quadratic Lyapunov function common for a whole class of such systems described using a quadratic form? Subsequently, its fundamental character was manifested in the discovery and effective utilization of its connections with a number of other fields. The theory of optimal control was among the historically first of them. Here, the frequency theorem proved to be a powerful constructive tool for checking the solvability of linear quadratic control problems (a combination of a linear control system and a quadratic performance criterion) and designing their solutions in the engineeringly attractive form of an optimal controller.

The foundations of the linear quadratic theory of optimal control were laid by the classical works of R. Kalman [78], N.N. Krasovskii [79], and A.M. Letov [80] (in the part concerning stochastic objects, by the investigations of A.N. Kolmogorov [81], N. Wiener [82], and S. Bucy [83]); a significant contribution to its development was made by J.C. Willems, V.I. Zubov, V.M. Kuntsevich, A.B. Kurzhanski, J.-L. Lions, A.I. Lurie, V.I. Utkin, V.A. Yakubovich, and many other scientists. (For the history of the linear quadratic theory of optimal control, see the surveys [84, 85].) Methodologically, this field has important connections with complex analysis, stability, and stabilization theory of nonlinear dynamic systems ([77, 86–88], etc.). First of all, it concerns the so-called uncertain systems, where the epithet reflects a common situation for applications: complete information about the system is unavailable. Starting from the late 1960s, the flow of scientific publications on the subject has become an avalanche, with a persistent marked interest until the present time. One reason is the generally recognized practical effect of the linear quadratic theory of optimal control. For example, according to the plenary report of Prof. M. Morari at the Second European Control Conference (Groningen, the Netherlands, 1993) [89], the linear quadratic theory occupies an honorable second place in the intensity of use in civil industrial applications among all branches of modern mathematical control theory. (The first place was given to the theory of PID controllers.)

In the theory of linear-quadratic optimization, the school of V.A. Yakubovich systematically developed the approach based on the frequency theorem. In the works of V.A. Yakubovich, A.I. Shepeljavyi, A.L. Likhtarnikov, A.V. Megretsky, S.G. Semenov, D.V. Plyako, A.V. Savkin, etc., the approach was extended to a wide class of important problems and systems, including, among others, systems with continuous and discrete time, systems with infinite-dimensional state space, problems arising under conflict (differential games), and problems with the singularity effect [85], which may cause no solution in the conventional sense.

The frequency theorem as a criterion for the existence of a quadratic Lyapunov function is traditionally and often supplemented by a special technique for constructing such a function, the so-called S-procedure [90]. In [91], it was abstracted from Lyapunov functions and given the sense of replacing (in a certain interpretation) a system of several inequalities by a single inequality with a free parameter. The key question here consists in the following: is the replacement equivalent? Under the affirmative answer, the S-procedure is said to be lossless, and the corresponding statement is also called the S-lemma. This question is relevant to several fields of mathematics [77], e.g., duality in extremum problems, matrix theory, and operator theory. The case of quadratic inequalities, where the losslessness of the S-procedure adjoins the effect of hidden (non-obvious) convexity of images of quadratic mappings [66], has proved to be particularly productive for control theory. Classical results of this kind are the Dines theorem (two quadratic forms transform a real linear space into a convex set) and the Toeplitz–Hausdorff theorem (two continuous Hermite forms transform the sphere of a complex Hilbert space into a convex set).

The first studies of the School on the losslessness of the S-procedure [92, 93] started at the turn of the 1970s and dealt with no more than three inequalities. Basically, they stayed within the idea field of the Dines and Toeplitz–Hausdorff theorems, in which, according to P. Halmos [94], all known proofs are based on calculations, although it is desirable to have an idea proof, at least (or especially?) using less elementary concepts. Further research of the School on the subject can be interpreted as a movement in the above direction, where the main goal was generalization to an arbitrary number of forms (unattainable in the general case). A noticeable impetus to this research was given by the students of V.A. Yakubovich and N.K. Nikol'skii in the work [95], where the convexity of the joint image was established for an arbitrary number of forms but in a very special situation motivated by control theory. The specialization was rather quickly overcome by V.A. Yakubovich together with A.S. Matveev, who joined this subject a little later: they obtained a series of general results on the losslessness of the S-procedure and the hidden convexity of quadratic functionals. In their works, the less elementary common reason for convexity was the invariance of forms with respect to shift operators and the weak convergence to zero of shifted space elements (as, e.g., in $L_2(0,\infty)$ when shifted by $T\to\infty$) [96]. Those results concerned important problems in control theory but did not cover the classical Dines and Toeplitz–Hausdorff theorems. Subsequently, A.S. Matveev obtained even more general criteria for the convexity of the joint image of an arbitrary number of forms; they "automatically" covered the classical results and presented certain properties (obviously fulfilled in the classical case) of the peripheral part of the spectrum of the operator bundle generated by the forms as a "less elementary" reason of convexity [97–99]. In that series of papers, the theory of approximate convexity of images of quadratic mappings with defect estimates was also developed, the property of hyper-convexity was discovered and investigated, and the results were extended to more general (non-quadratic) mappings. Several results in this direction were also obtained by outstanding scientist B.T. Polyak [100, 101]. Motivated by the theory of stochastic control, N.G. Dokuchaev (with the participation of V.A. Yakubovich) developed a parallel ideology related to A.A. Lyapunov's effect (the convexity of the image of an atomless vector measure).

At the turn of the 2000s, an important discovery presented the relationship and interaction of the S-procedure and the frequency theorem in a completely new light. Namely, a new proof of the frequency theorem was given in [102] based on the losslessness theorem of the S-procedure (S-lemma). As a result, figuratively speaking [66], the frequency theorem and the S-procedure lived for a long time as friendly neighbors, and now, after so many years, everyone has found out that they are also relatives.

The work [102] stimulated research on the so-called generalized frequency theorem (generalized KYP-lemma), establishing applications-relevant properties equivalent to the fulfillment of frequency domain inequalities in some restricted frequency range. The corresponding results provide new system analysis and design tools related to frequency domain inequalities satisfied in a finite frequency range [102, 103]. As it turned out, the standard frequency domain inequality in a finite frequency range is equivalent to some non-classical linear matrix inequalities for the pair of matrices P, Q; in a certain sense, these inequalities are analogous to and "replace" the inequalities for a single matrix P in the classical KYP lemma. According to [104], the frequency domain inequality in a finite frequency range is, in turn, equivalent to definite inequalities (of the dissipation type) only on part of the system trajectories defined by an additional integral matrix inequality (the so-called restricted dissipativity [104]). Thus, a complete extension of the classical KYP results to the finite-frequency case was obtained. In [105], the above results were further generalized to the case of the "conic" S-procedure to work with an infinite number of constraints. The finite-frequency version of the frequency theorem has already found application in several practical problems [106–108].

In the 1990s, the three main scientific directions of the School—the frequency theorem, the S-procedure, and linear-quadratic optimization—merged in the research on nonconvex global optimization methods. More precisely, the matter concerns a general approach based on these directions in order to develop efficient algorithms for special problems in the field of nonconvex global optimization in a standard way. Unlike the majority of methods in this field, which are mostly computational, often involve heuristic ideas, and do not always converge, the algorithms mentioned above rest on a mathematical theory, are analytical in their most essential part, and surely yield the global optimum. The general approach was proposed by V.A. Yakubovich in 1992 [109, 110]. It was further developed in the works of V.A. Yakubovich, A.S. Matveev, and N.G. Dokuchaev. This approach justifies the basic relations of the theory of convex duality for the nonconvex optimization problems under consideration and solves them using the (Yakubovich) rule based on the relations. The rule is not necessarily correct. It was established that the rule is correct whenever it is effective (produces a non-empty set of answers). Despite this fact, of greatest interest are the criteria to verify the applicability of the rule a priori (before its application) based on a (usually simple) check of certain properties of the initial problem data. Several such criteria were derived. Note that in many respects, these criteria served as the main purpose of the studies of images of quadratic mappings discussed above.

In the late 1970s, V.A. Yakubovich initiated an extensive cycle of research for his team to elaborate the theory of the maximum principle in optimal control problems within an abstract approach. This approach implies the choice of some abstract model described by the language of functional analysis as the main object of study. The results obtained for the model are then supposed to be interpreted with respect to the specific models encountered by the researcher. Thus, when working with a variety of applications, this approach allows reducing the amount of considerations: much of them have already been done once and for all within the abstract theory. Another advantage of the abstract approach is a uniform procedure for deriving optimality conditions. Methodologically, it provides a more accessible and simple presentation of the main ideas: they are not obscured by the entourage of a particular model.

Such abstract theories were elaborated by many authors. V.A. Yakubovich proposed his own (original) approach to constructing an abstract theory of optimal control. Its characteristic feature consists in the apparatus of the calculus of differentials on bundles of (generally nondifferentiable) curves. On this basis, an abstract maximum principle is established for an abstract model of an optimal control problem. In particular, it explains why maximum principles analogous to Pontryagin's maximum principle naturally arise as necessary conditions of optimality in very seemingly different problems, highlighting the general properties of the problem that predetermine the specified form of the answer. This approach also yielded a uniform theory of necessary conditions of the first and higher orders in problems with constraints: all of them turn out to be parts of some single condition [111]. The approach under discussion was developed in different directions in an extensive series of works by V.A. Yakubovich and his students; a number of self-sufficient new results were obtained on its basis. Some of them (e.g., concerning the optimal control of systems described by partial differential equations) were significantly ahead of similar research results in the world. Some outcomes of those studies were systematized in the books by A.S. Matveev and V.A. Yakubovich [111, 112]. The textbook [112] was intended to teach the reader to independently apply the abstract theory to new problems. The book contains 75 problems on the application of this theory. Some of them correspond to the level of scientific publications of the recent past; at the same time, they are successfully handled by fourth-year students of the Faculty of Mathematics and Mechanics (SPbSU).

Since the inception of LTC and DTC, there were two assistant captains on their command bridge: A.Kh. Gelig and V.N. Fomin. The main interests of A.Kh. Gelig were focused on analyzing the dynamics of different types of pulse-modulated systems. In this area, he developed a new approach based on the time-averaging of the pulse signal and the absolute stability theory of continuous nonlinear systems. Unlike the classical averaging method, Gelig's averaging is not asymptotic in nature and allows estimating the required sampling frequency explicitly. Classical theorems of the absolute stability theory of nonlinear systems (such as the famous circle criterion and V.M. Popov's criterion, as well as the stability criteria of periodic modes) are obtained as limiting cases when the value of the discretization period tends to zero. Therefore, the constructed theory has a high degree of unification. The corresponding cycle of works was summarized in the joint monograph by A.Kh. Gelig and A.N. Churilov, first in Russian and then in English [113] (the extended version published by Birkhauser). A.Kh. Gelig's long-term interests also included the analytical design of controllers for nonlinear systems. In contact with his many-year collaborators I.E. Zuber and A.N. Churilov, he solved various stability and stabilization problems for continuous, pulse-modulated, and discrete systems in the cases of state- and output-feedback control [114]. A.Kh. Gelig was among the pioneers investigating nonlinear dynamics of neural networks in the

USSR [115]; together with V.A. Yakubovich and G.A. Leonov, he studied the stability of systems with a nonunique equilibrium (stationary sets) [48].

V.N. Fomin began his scientific activity with the study of parametric resonance in Hamiltonian systems described by partial differential equations. Here, he managed to construct a rather complete analog of the finite-dimensional theory based on Galerkin's method and a variant of the latter's perturbation method. After defending his doctoral dissertation on this subject in 1971, V.N. Fomin's research interests shifted to the field of mathematical theory of cybernetic systems. He paid special attention to topics related to machine learning and adaptive systems, demonstrating an encyclopedic coverage of the subject. His monograph [116] and the coherent course of lectures were among the first in the country on these very important topics and painted a broad picture of the field, not limited to a single group or approach. In 1976, the book [116] was awarded the first prize of LSU in the field of scientific works. The third main direction of V.N. Fomin's work gradually gained strength: the mathematical theory of filtering and control theory, first of all, in its probabilistic variant [117–119]. Here, he obtained numerous results concerning, among others, the stochastic linear-quadratic optimal control problem, spectral factorization, and optimal estimation of random processes and fields; he developed methods for designing optimal filters when processing a packet of random plane waves against the background of distributed noise. The results of this cycle have important applications in the theory of radar and short-wave communications, underwater acoustics, radio astronomy, seismology, geophysics, and television tracking systems. The tendency to use the power of functional analysis in control theory, general for the school of V.A. Yakubovich, did not pass over V.N. Fomin. In recent years, before his untimely death, he actively and passionately developed the operator approach to filtering problems and related control problems. In particular, he succeeded in constructing a unified theory of optimal filtering, which effortlessly encompasses the Wiener–Kolmogorov theory of optimal filtering of stationary processes and the Kalman–Bucy theory of recursive filtering and, moreover, has a wide scope of applicability. Vladimir Nikolaevich's energy, charisma, and sparkling humor made him the driver of almost any event (seminar, lecture, etc.) with his participation, and the main claim of students who were lucky enough to attend his lectures was that they could never fall asleep.

In 1969, a new postgraduate—G.A. Leonov—appeared in LTC. In 1971, he defended his candidate's dissertation and continued his work in the Laboratory and at the Department. Gradually, an individual scientific direction was formed under his leadership within the traditional LTC–DTC approaches. The fundamental results on the theory of stability and synchronization of nonlinear oscillations in phase systems [48, 51, 120, 121] were followed by pioneering works and books on the theory of control and stabilization of linear controlled systems [122, 123] and the qualitative study of global attractors in dynamic systems: instability, bifurcations, synchronization, and dimension estimation [124, 125]. In 2007, G.A. Leonov became Head of the Department of Applied Cybernetics, newly established at the Faculty of Mathematics and Mechanics (SPbSU), and subsequently part of its history.

In V.A. Yakubovich and the older generation of his students, a keen interest in practical problems was naturally combined with the I. Kant's thesis that there is as much truth in each science as there is substantial mathematics in it. Among the next generation, a bright adherent of this philosophy was A.E. Barabanov, a student of V.N. Fomin. Colleagues repeatedly admired Andrei Evgen'evich's ability to apply deeply non-trivial mathematical moves in seemingly routine but important applied problems. And, more significantly, it brought success, confirming the above thesis. The range of A.E. Barabanov's interests was vast. As an illustration, let us mention important R&D works for the defense industry, the development of interference-proof dial-up modems for highly noisy switched lines (together with employees of the Department of System Programming, SPbSU), and radar signal processing systems, first for NPO Ravenstvo and then for Transas, one of the world's largest suppliers of maritime navigation software. (According to experts, e.g., A.N. Terekhov[2], Transas was a monopolist of the onboard software market in the 1990s–2010s.) Note also systems for analyzing dolphin sound signals and systems for speech analysis and synthesis, which were developed in creative contact with the Department of Phonetics, SPbSU. On the latter topics, A.E. Barabanov prepared and delivered advanced courses of lectures. The focus of his theoretical research was on the design of optimal and suboptimal controllers [126, 127], where he obtained a series of important and sometimes unexpected results. As an example, in the 1980s he designed an optimal controller under uniformly bounded perturbations and, on this basis, constructed a new theory of L_1-optimal control. The pioneering work of A.E. Barabanov and O.N. Granichin on this topic was far ahead of similar foreign publications. Later on, O.N. Granichin (another student of V.N. Fomin) systematically developed approaches based on randomization in control systems and obtained the conditions of system operability under "almost arbitrary" (unknown but bounded) disturbances [128, 129].

At the turn of the millennium, the scientific community realized that, on the one hand, a continuous physical process interacting with a discrete (digital) control computing device is a steadily spreading combination of the future; on the other hand, the available tools of the mathematical theory are not ready, to the extent required, to deal with this combination. Its mathematical model is the hybrid dynamic system (HDS), i.e., a system described by both continuous and discrete state variables that mutually affect the evolution of each other. In the late 1990s, the interest in the mathematical modeling and theory of such systems could be characterized as a kind of boom.

Since 1997 the students of V.A. Yakubovich—A.S. Matveev and A.V. Savkin—conducted joint studies on the qualitative theory of HDSs. They laid the foundations of such a theory for a rather general class of HDSs and obtained some of the first general proof results in this field. The results were published in leading international journals, as well as in the monograph [130], probably the first in the world on this subject. The corresponding series of works focused on a general class of switched HDSs, i.e., systems for which the continuous state variables have no jumps. Among other things, the outcomes include necessary and sufficient conditions of strong determinacy of the system and invariance of a given domain, criteria for the existence and global stability of limit cycles, analogs of the classical Poincaré–Bendixson theorem, a method for designing distributed switching algorithms for processors ensuring excitation and global stability of given (optimal) oscillatory processes in large-scale flow networks, etc. The effectiveness of the general theory was demonstrated by the productive study of a number of models of information, computer, transportation, and other networks, flexible manufacturing systems, biotechnological, and other processes of independent interest.

Various aspects of the discretization problem of continuous systems in the general context of constructing the theory of HDSs were actively studied by V.A. Bondarko [131, 132]. For example, for linear time-invariant objects, he compared different discretization methods in terms of their adequacy, interconnections, and the asymptotics of properties of discrete models when increasing the frequency of time quantization. In parallel with refining the theory of finitely convergent algorithms for solving countable systems of inequalities, V.A. Bondarko established important results in the field of adaptive control, including the control of nonlinear systems and systems with an infinite-dimensional state space.

Since the mid-1990s V.A. Yakubovich and his students (with a special role of A.V. Proskurnikov among them) breathed new life into the traditional topics of the School related to linear-quadratic optimization and the frequency theorem: a cycle of about 20 papers was published on optimal damping of oscillations, optimal signal tracking, and invariance theory [133–138]. Within this cycle, a number of pioneering aspects were introduced into quite classical control theory topics, including

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[2] `https://www.rbc.ru/spb{\_}sz/22/03/2018/5ab26f809a7947027cb81160`.

the conceptualization of a "universal controller" ensuring optimality under all a priori unknown noises and tracked signals and the invariance of the system output with respect to the exogenous disturbance. The seminal paper [133] of the cycle won the Nauka/Interperiodica's award for the best publication of the year 1995; at the 1995 European Control Conference, V.A. Yakubovich was a plenary speaker on this subject [134]. In 2008, A.V. Proskurnikov was awarded the Young Scientists Medal and Prize of the Russian Academy of Sciences for for the cycle under discussion.

Starting from about the early 2000s, the phenomenon of swarm intelligence in complex network systems, as it is now called, has attracted considerable interest from physicists, mathematicians, and computer scientists in the world. Here, the main intrigue lies in how the local interactions of uninformed and low-influence elements give birth to rational and meaningful behavior of the network as a whole. The motivation for this topic is diverse and includes investigating the dynamics of ensembles of physical particles, biological populations, and opinions in social groups, artificial intelligence systems, networked control systems, etc. The subject of one of the most mathematically substantial (to date) sections of this field is distributed consensus algorithms. Significant results of the school of V.A. Yakubovich in this field, unfortunately, still poorly represented in the Russian Federation, belong to A.V. Proskurnikov (with the initial participation of A.S. Matveev); in 2022, he defended his doctoral dissertation on this topic at SPbSU. It crowns the cycle of studies, in particular, with a remarkable advance toward a complete theory of distributed averaging consensus algorithms and a productive original method of differential and recursive averaging inequalities. A.V. Proskurnikov's contribution to the related development of mathematical sociology was recognized by a joint publication in *Science* [139].

Among the examples of initiative work by V.A. Yakubovich's students, it is necessary to mention the development and promotion of a new and, to a large extent, pioneering direction lying at the junction of physics and cybernetics. This direction emerged not by chance; on the contrary, at the turn of the 1990s, there was an explosive interest in the application of cybernetics and information and control theory methods in physics. One of its triggers was the intriguing possibility, discovered in those years, to significantly change the properties of a system, e.g., to suppress or create chaos in its behavior, to change its resonance characteristics, etc., through a (theoretically arbitrarily) small impact. The books [140, 141] published in 1998 and 1999 were the first monographs in the world in this direction, and the field of sciences on the border of cybernetics and physics was named cybernetical physics (cyber-physics) by their coauthor A.L. Fradkov. In particular, it includes the control of molecular and quantum systems (A.L. Fradkov, M.S. Anan'evskii), which play an important role in the creation of promising nanotechnologies. The paper [142] reviewing research works on chaos control[3] won the Nauka/Interperiodica's award for the best publication of the year 2003. The basic principles of cyber-physics were described in the books [42, 143]. Signs of its international recognition are the world's first international conferences on physics and control held in St. Petersburg (2003–2005), as well as the International Physics and Control Society (IPACS) established with headquarters in St. Petersburg. *Cybernetics and Physics*, an international journal indexed in Scopus, is published in St. Petersburg under the auspices of this society.

Another important direction, perhaps decisive for cybernetics itself, reflects the convergence trend of the theories of control, computation, and communication toward their unity, which took shape at the turn of the millennium. More and more problems require close interaction of the methods of these three theories; even the aphoristic formula Control×Computation×Communication= C^3 has appeared, expressing the aspiration to return the holistic perception of information, computational, and control processes, which meant so much for the successes of the "romantic" cybernetics of the 1960s. Some pioneering results in this direction were obtained in the early 2000s by

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[3] The survey is the most cited article of *Avtomatika i Telemekhanika*, and its second co-author is the most cited author of the journal.

A.S. Matveev together with A.V. Savkin, a DTC alumnus and Professor of the University of New South Wales (Australia). The corresponding cycle of works was devoted to control and estimation under the capacity constraints of communication channels [144–147] and was partially summarized in the monograph [148]. In particular, it was demonstrated that an unstable linear controlled system can be stabilized if and only if the bit rate of information arrival through the communication channel exceeds the rate of information production by the system; also, a fundamental advance was made to determine the place of the basic concepts of C. Shannon's information theory (in particular, the capacity of a noisy communication channel) in the discussed topic. Subsequently, the research switched to nonlinear systems and was carried out by A.S. Matveev in coauthorship with Professor A.Yu. Pogromsky (the Eindhoven University of Technology) using the nonlinear dynamics analysis methods of A.M. Lyapunov and G.A. Leonov. In particular, a new concept of the restoration entropy of a nonlinear system was developed; in co-authorship with C. Kawan (Ludwig-Maximilians-Universitä München) it was shown that, in a certain sense and in the questions under consideration, this entropy adequately characterizes the rate of information production by the system [149, 150]. Sufficient conditions for the operability of nonlinear and adaptive systems under communication constraints were also obtained by A.L. Fradkov et al. [151–156]. Some results obtained by the team were overviewed in [157].

With all the conventionality of any rubric, it has become a kind of tradition to divide modern robotics into two sections, industrial and mobile robotics. The first (and more developed) section focuses on the orchestra of industrial systems, in which manipulation systems (mechanical arms) play first fiddle. At present, the vast majority of such systems follow the hold-down arm paradigm with a fixed operational location. At the same time, practical tasks are systematically introduced into the agenda where the soft (non-hold-down arm) approach is needed and/or the manipulation object is malleable, and mobile and manipulation functions interact operatively. Such tasks now fall into an almost unexplored field; its development requires solving a number of fundamental problems, including theoretical ones. A group of DTC graduates (A.S. Shiryaev and S.V. Gusev) has been systematically working in this direction since the 2010s. Its asset is the development of largely pioneering mathematical methods of dynamics analysis and controller design for solving the corresponding problems, in particular, the method of moving Poincaré sections and transverse linearization, high-speed methods for solving special matrix Riccati differential equations, general methods for finding periodic motions implemented in complex under-actuated mechanical systems, and other results [158–160]. The effectiveness of these R&D results was demonstrated in 2015 by the world's first experimentally validated solution of a complex prototypical problem posed in 1998 by C. Lynch: stabilize the circular motion of a ball on a rotating butterfly-shaped guide [160].[4]

The mobile robotics section concentrates on the autonomous navigation of mobile robots and their motion control in a priori unknown environments with obstacles. This direction has been systematically developed since the 2010s by the mobile robotics group of DTC (A.S. Matveev, A.A. Semakova, and P.A. Konovalov) with the participation of A.V. Savkin (until 2017). A number of fundamental results on robot navigation algorithms in complex (particularly moving and unpredictable) environments, including distributed control of their multi-agent ensembles, were obtained here. They were partially systematized in the two monographs [161, 162], released in 2015 and 2016 by the world's leading academic publishers. The specifics of the group's R&D works are resource-saving algorithms (in terms of computations, energy, sensory data about the environment, etc.) that convert current observation into current control in a reflex-like manner (as a consequence, with minimal requirements for onboard processors) and are nevertheless provided with mathematically rigorous guarantees of achieving the result. According to the WoS data for the year 2022,

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[4] https://www.youtube.com/watch?v=kyvW5sOcZHU.

of the five most cited publications on robotics affiliated with Russia, four are related to DTC, including the most cited paper [163] (279 citations).

Mathematical methods have long been used for the quantitative and qualitative study of processes and systems, to a greater or lesser extent related to the field of biology and medicine. In this direction, in the early 2000s A.S. Matveev and A.V. Savkin investigated optimal protocols for chemotherapeutic treatment of cancer [164]. Starting from the mid-2000s, DTC (A.N. Churilov and A.I. Shepeljavyi) together with Uppsala University (Sweden) conducted systematic studies on modeling and analysis of biological rhythms and chaotic dynamics in neurohormonal systems [165, 166]. Since the 2010s the scientific directions of the School include neural control and neurofeedback based on the mathematical study of networks of biological neurons. These R&D works lie at the junction of cybernetics and neuroscience; here, the world expects breakthroughs in medical diagnosis, as well as in the control of robots and other devices with the power of thought (without human muscles). At present, under the guidance of A.L. Fradkov, a grant-supported project is being implemented on this topic at SPbSU. The corresponding works are being carried out jointly with the Higher Nervous Activity and Psychophysiology Department of SPbSU, the Institute of Human Brain (the Russian Academy of Sciences), Institute for Problems of Mechanical Engineering (the Russian Academy of Sciences), and Immanuel Kant Baltic Federal University. M. Lipkovich and S.A. Plotnikov, young representatives of the School, actively participate in the project.

Representatives of the School have been teaching at various universities of the country. In St. Petersburg, let us note the following persons (currently active or passed away): G.A. Leonov, Dean of the Faculty of Mathematics and Mechanics (SPbSU), USSR State Prize Laureate, Corresponding Member of the Russian Academy of Sciences; N.V. Kuznetsov, Head of the Department of Applied Cybernetics (SPbSU), Corresponding Member of the Russian Academy of Sciences; O.N. Granichin, Professor of the Department of System Programming (SPbSU); Professors A.V. Timofeev (St. Petersburg State University of Aerospace Instrumentation), A.N. Churilov (St. Petersburg State Marine Technical University), V.B. Smirnova (St. Petersburg State University of Architecture and Civil Engineering), N.E. Barabanov (St. Petersburg Electrotechnical University "LETI"); Heads of laboratories of academic institutes A.V. Timofeev (St. Petersburg Institute for Informatics and Automation, the Russian Academy of Sciences) and A.L. Fradkov (Institute for Problems of Mechanical Engineering, the Russian Academy of Sciences). In the 1970s and 1990s, several talented graduates of the Department left the country, B.G. Pittel, M.V. Levit, and B.D. Lyubachevskii were among them. Some of them became professors at foreign universities: A. Megretski (Massachusetts Institute of Technology, USA), N. Barabanov (North Dakota State University, USA), A. Savkin (University of New South Wales, Australia), A. Shiriaev (Umeå University, Sweden, and Norwegian University of Science and Technology, Trondheim, Norway)

A significant place in the School's activities is occupied by scientific and organizational work. For example, since 1967 V.A. Yakubovich was Deputy Chairman (Deputy of A.A. Vavilov, Rector of Leningrad Electrotechnical University) and part-time Chairman of the Section for the Theory of Adaptive Control Systems, the Leningrad Territorial Group of the National Committee on Automatic Control. A series of six Leningrad (St. Petersburg) symposia and one All-Union Conference on the Theory of Adaptive Systems, held on the initiative of V.A. Yakubovich and under his guidance from 1972 to 1999, occupied a notable place in the scientific and organizational landscape of the country. This series was another sign recognizing the School's merits in the field of adaptive systems, and its events were important milestones in the development of the field. In those years, it was one of the main growth points of mathematical control theory and cybernetics and attracted the interest of talented young people and venerable researchers: the number of papers and participants usually numbered in the hundreds. The symposia were attended by leaders of domestic and, since the 1990s, foreign science. Note the following persons among them: Academicians Ya.Z Tsypkin, A.A. Krasovskii, E.P. Popov, and N.N. Moiseev; Doctors of Science D.A. Pospelov, V.Yu. Rutkovskii, Yu.I. Neimark, A.A. Pervozvanskii, and R.M. Yusupov; G. Bartolini (Italy), S. Bittanti (Italy), V. Răsvan (Romania), A. Halanay (Romania), L. Ljung (Sweden), J. Lando (France), A. Lindqvist (Sweden), D. Šiljak (USA), K. Furuta (Japan), and others. In 1972, a plenary report was delivered by M.M. Botvinnik, Doctor of Engineering, former world chess champion; he spoke about the development of a computer algorithm for chess play. Initially, the scientific secretary of the series of events was D.P. Derevitskii, Associate Professor of the Department of Automatic Control Systems (Leningrad Mechanical Institute); later, he was replaced by A.L. Fradkov. In scientific and organizational activities, DTC traditionally and closely cooperates with the Laboratory of Complex Systems Control (Institute for Problems of Mechanical Engineering, the Russian Academy of Sciences). It was established in 1990 by A.L. Fradkov, the first and present-day head. This laboratory is closely connected with the Department both in research interests and in education.

The team conducts career-oriented work with young people in the field of cybernetics. In 1999, a group of experts in automation and control systems from several universities of the city proposed to organize school olympiads in cybernetics. The idea was supported by V.P. Tarasov, Head of the department of science and technology at St. Petersburg City Palace of Youth Creativity (the Anichkov Palace), and things got rolling: 14 Olympiads were held in 1999–2013. M.S. Anan'evskii, A.L. Fradkov, and A.S. Matveev, representatives of the School, took an active part in their organization and holding from the very beginning. The materials of the Olympiads and some methodological conclusions were summarized in a series of proceedings published largely owing to the work and energy of M.S. Anan'evskii.

In 2008, a cybernetics club was organized for junior students of DTC based on LEGO Mindstorms NXT. While learning control theory, students had an opportunity to implement control algorithms on physical objects and to connect their theoretical knowledge with practice. In the class, students independently developed original designs such as a bicycle robot, a segway, a crawling robot, a predator robot, and others. The best works were presented at the Robot Show during the Week of the Faculty of Mathematics and Mechanics (SPbSU). At the same time, creative cooperation began with the Robotics Center of Presidential Physics and Mathematics Lyceum (PPML) No. 239, headed by S.A. Filippov. The results of cooperation were presented at several international conferences [167, 168]. The enthusiasm of R.M. Luchin, a DTC member and teacher, played an important role in organizing and leading the club. Robot soccer became one direction of his work. The first city competitions of radio-controlled robots were held in 2012; a year later autonomous robots were already on the field.

Experience with LEGO led a group of enthusiasts (R.M. Luchin, S.A. Filippov, and A.N. Terekhov) to the idea of developing their own constructor set, more advanced than LEGO. Cybernetic Technologies LLC was founded, where the Universal Cybernetic Constructor TRIK and the necessary software were developed, allowing to implement various projects, from basic educational to modern research projects. They are used in Russian schools and universities. According to the 2018 annual analytical review of the global robotics market by Sberbank's robotics laboratory, the company was mentioned as one of Russia's few unconditional successes in this market so far. Unfortunately, R.M. Luchin passed away prematurely at a young age due to the COVID-19 pandemic. His work is being continued by his student, I.Yu Shirokolobov, an employee of DTC. In 2019, URoboRus, the jointly created team of robotic soccer players, was the first Russian team to qualify for the RoboCup SSL, a kind of world championship. In 2020, URoboRus qualified again, but the competition was canceled due to the pandemic. In 2021, the competition was carried out online, and URoboRus managed to participate in the playoff for the first time, ranking first in the group. The year 2022 was remarkable for another successful qualification for the RoboCup

SSL World Championship and the first full-time participation. The event took place at the FEI University in Sao Paolo (Brazil) during RoboCup Brazil Open, the Brazilian Open Championship.

Thanks to the efforts of the DTC team, as well as the support of PPML No. 239, the Scientific and Educational Center (SEC) "Mathematical Robotics and Artificial Intelligence" was established at SPbSU in 2019. Since its foundation, K.S. Amelin (a student of O.N. Granichin) and A.L. Fradkov are Director and Scientific Supervisor of SEC, respectively. The Center is intended to integrate the efforts of SPbSU in fundamental research on mathematical and educational robotics and intelligent control. The directions of the Center's work include the issues of navigation of mobile robots and their multi-agent network ensembles, control of underactuated manipulators, computer vision, artificial intelligence, machine learning and big data processing, neural network control, methods and tools for programming and debugging robots, and educational and practical robotics. In 2022, the experimental park of SEC contained Geoscan Pioneer quadcopters and TRIK universal cybernetic constructor kits. With the active participation of SPbSU students and the use of this park, SEC has already implemented several applied projects, in particular, forrest inventory by robotic quadcopters, search for a person lost in the forest, semi-automatic dropping of GPS beacons on glaciers to monitor their movement, automatic overflight of protected areas, control of bridge piers, increase of the data transmission rate in large wireless networks, etc.

Additional information about the department is available in thematic issues of Russian and international journals [169–171] dedicated to the anniversaries of DTC employees and in the collection of articles [3]. The scientific product of the School counts many hundreds of publications, including over 60 books. V.A. Yakubovich's nestlings work fruitfully in many Russian and foreign research centers and universities; they have defended over 100 dissertations on physics, mathematics, and engineering, including 19 doctoral dissertations.

The influence of the Department and School's achievements is noticeable in the distribution of university places in world rankings. For example, according to the Shanghai Academic Ranking of World Universities (ARWU), SPbSU ranked 32nd in the direction "Automation and Control" in 2018. The number of publications by DTC employees in top journals on automation and control, taken into account in the ARWU ranking, approximately equals 28% (24 out of 85) of all Russian publications in such journals for 2012–2016.

Representatives of the School have been repeatedly given prestigious Russian and international awards and titles. In 1998, V.A. Yakubovich became Honored Scientist of the Russian Federation; in 2005, he was awarded the Order of Honor. V.A. Yakubovich was Member of the Russian Academy of Sciences and Academician of the Russian Academy of Natural Sciences. In 2006 he was elected Honorary Professor of SPbSU. A.L. Fradkov was awarded the international honorary titles of IFAC Fellow, IEEE Life Fellow, and AAIA Fellow. In 2015, DTC alumnus (1998) Alexey Pavlov and colleagues from the Eindhoven University of Technology received the prestigious IEEE Control Systems Technology award. In 2020, DTC alumnus A.V. Proskurnikov and coauthors were awarded the IFAC and Elsevier paper prize award for the best paper published in *Annual Reviews in Control* in 2017–2020. (Proskurnikov, A.V. and Tempo, R., A Tutorial on Modeling and Analysis of Dynamic Social Networks. Part I, *Annual Reviews in Control*, 2017, vol. 43, pp. 65–79.) The same award for the best paper of 2020–2022 was given for the survey [76]. In 2018, A.L. Fradkov was awarded the Andronov Prize of the Russian Academy of Sciences for the series of works on synchronization and control of nonlinear oscillations (together with I.I. Blekhman).

After the demise of Vladimir Andreyevich Yakubovich in 2012, the founder and long-term head of DTC, the founder of the scientific school of cybernetics and artificial intelligence in St. Petersburg, the Department was successively headed by his closest colleagues and students, A.Kh. Gelig, A.L. Fradkov, and A.S. Matveev (since 2021 until present). Several thematic collections, publications, and speeches have been devoted to the creative biography and scientific achievements of

V.A. Yakubovich, as well as the 1st International IFAC Conference on Modelling, Identification and Control of Nonlinear Systems (MICNON 2015) [172–174]. The DTC staff prepared and published a CD-ROM containing over 300 main works of V.A. Yakubovich.

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